Bose-Einstein condensates and antiferromagnetic interactions

An illustration of symmetry breaking

Jean Dalibard
Collège de France and Laboratoire Kastler Brossel

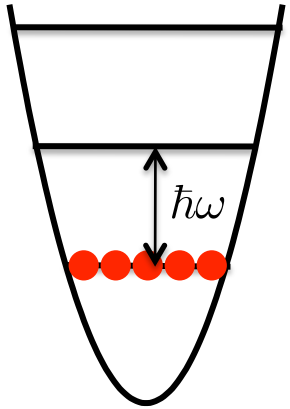
The simplest many-body system

Quantum fluid with N bosons

Continuous degrees of freedom	Discrete degrees of freedom
$\vec{r}_i, \ \vec{p}_i$	$\vec{s}_i$
infinite Hilbert space	*finite Hilbert space*

We will look for a situation where the external degrees of freedom are frozen

To first approximation, all atoms occupy the ground state of a tight laser trap



$k_B T \ll \hbar\omega$

Only the spin degrees of freedom remain relevant (Single Mode Approximation = SMA):

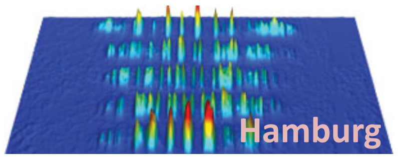
Corresponding interactions: $V_{\text{spin}} = \alpha \sum_{i<j} \vec{s}_i \cdot \vec{s}_j$ $\qquad$ $\alpha > 0$: antiferromagnetic

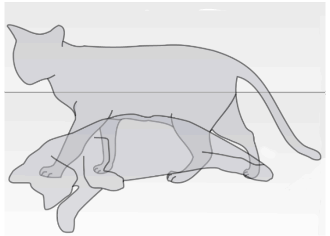
Motivations for spinor physics (also beyond SMA)

Stamper-Kurn & Ueda
Rev. Mod. Phys. (2013)

Coherent spin oscillation, spin mixing, dynamical instabilities

Georgia Tech, Hamburg, Hannover, Mainz-Munich, NIST, ...



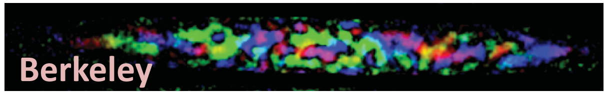


Spin squeezing & entanglement

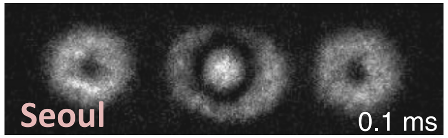
Georgia Tech, Hannover, Heidelberg, ...

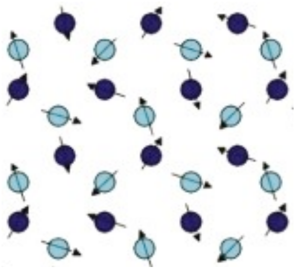
Quenched dynamics and pre-thermalization phenomena

Berkeley, Georgia Tech, Hamburg,...



Topological defects Boulder, MIT, Rochester, Seoul, ...





Model for Heisenberg spin lattice systems

Dipolar gases

Stuttgart, Hamburg, Innsbruck, Paris-Nord, Boulder, Stanford, ...

How to get non-trivial thermodynamics within SMA

$$H = \alpha \sum_{i<j} \vec{s}_i \cdot \vec{s}_j + \text{Zeeman effect}$$

$\vec{B}$ parallel to z

Constraint: conservation of magnetisation

Consider the total spin $\vec{S} = \sum_i \vec{s}_i$

Then : $[H, S_z] = 0$

- For an assembly of (effective) spins ½, once the initial numbers of $|s_z = +\frac{1}{2}\rangle$ and $|s_z = -\frac{1}{2}\rangle$ are known, nothing can happen

- For an assembly of spins 1, an interesting dynamics can still take place thanks to

$$|s_z = 0\rangle + |s_z = 0\rangle \quad \longleftrightarrow \quad |s_z = +1\rangle + |s_z = -1\rangle$$

Our setup: a BEC of ^{23}Na atoms in their $F = 1$ hyperfine state

Example: Ground state in the absence of external magnetic field $H = \frac{\alpha}{2}\vec{S}^2 + \text{constant}$

For $\alpha > 0$, the ground state is the singlet state $|S = 0\rangle$ (total spin zero), which corresponds to a strongly entangled state of the N spins 1

Law, Pu, Bigelow (1998)
Castin & Herzog (2000)
Ho & Yip (2000)

Outline of the talk

1. Experimental setup and control of magnetisation

2. Phase diagram of a BEC with anti-ferromagnetic interaction (within SMA)

3. The ground state of an anti-ferromagnetic BEC:
   A paradigm example of symmetry breaking in Quantum Mechanics

The ENS team

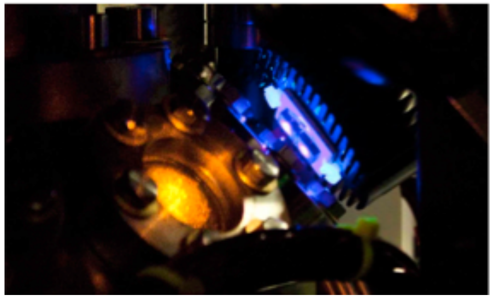
PhD students: Vincent Corre, Camille Frapolli, Lingxuan Shao

Postdoc: Tilman Zibold

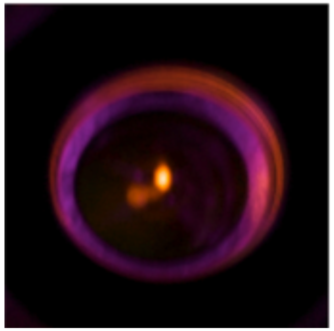
Permanent: Fabrice Gerbier, Jean Dalibard

Former members: Luigi De Sarlo, Emmanuel Mimoun, David Jacob

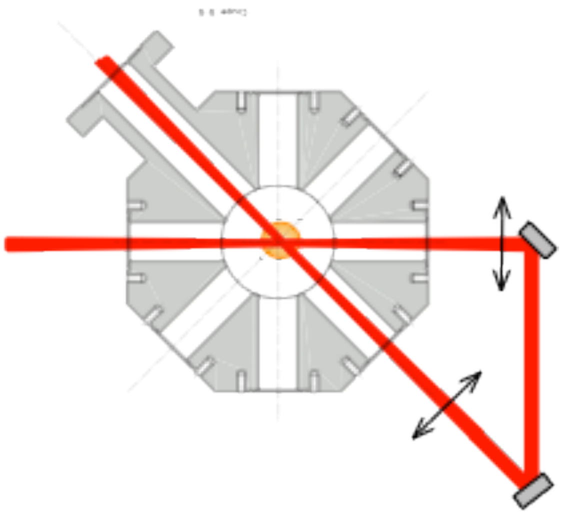
Sodium BEC in F=1 hyperfine state: Preparation of the sample



Room temperature vapour cell of Sodium
(using UV light-induced desorption)



Magneto-optical trap in the vapour cell



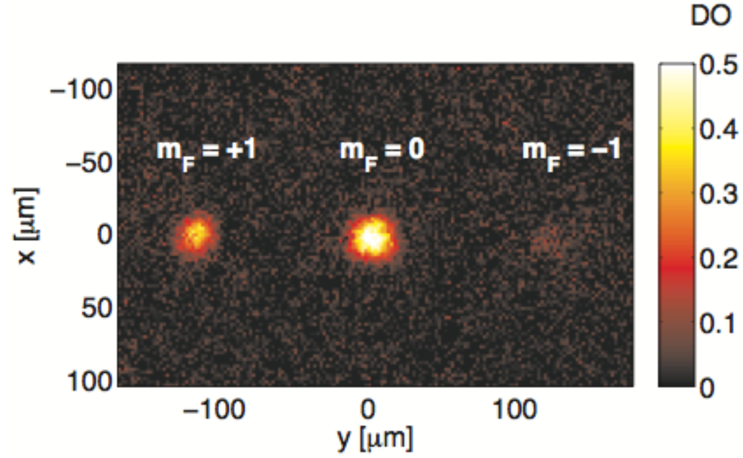
Evaporation and Bose-Einstein condensation in the $F = 1$ ground state in a crossed dipole trap + dimple

$$\omega/2\pi \approx 1 \text{ kHz}$$

quasi-pure BEC with 6000 atoms

Between ideal gas BEC and Thomas-Fermi regime: $E_{\text{int}} \approx \hbar\omega$

Magnetisation: Detection and control



Diagnostic of the sample by Stern-Gerlach analysis

Check of the single-mode approximation:
Same spatial profile for $s_z = -1, 0, +1$

In the absence of any specific procedure, the magnetisation of the sample is

$$M_z = \frac{N_{+1} - N_{-1}}{N_{+1} + N_0 + N_{-1}} \approx 0.5$$

(optical pumping, evaporation, ...)

We can decrease or increase this value at will:

Depolarization with a radio-frequency magnetic field

Lowers M_z to any adjustable value between 0.5 and 0

Spin distillation through evaporation in the presence of a magnetic gradient

Raises M_z up to 0.98

Effective anti-ferromagnetic interactions

Real magnetic interactions (dipole-dipole) are negligible at our temperature scale
Only van der Waals contact interactions play a significant role

For a collision between two spin 1 atoms, the total spin can be:

$S = 0$ (symmetric spin state)
$S = 1$ (anti-symmetric spin state)
$S = 2$ (symmetric spin state)

Ho (1998)
Ohmi & Machida (1998)

Since the orbital state is symmetric (all atoms in the same spatial mode), only the $S = 0$ and $S = 2$ channels are relevant $\longrightarrow$ two scattering lengths a_0 and a_2

$$V_{1,2} = \delta(\vec{r}_1 - \vec{r}_2)\,[\bar{g} + g_s \vec{s}_1 \cdot \vec{s}_2]$$

$$\bar{g} \propto \frac{a_0 + 2a_2}{3}$$

$$g_s \propto \frac{a_2 - a_0}{3}$$

Na: $g_s > 0$ (antiferro)
Rb: $g_s < 0$ (ferro)

Leads at the many-body level to $V_{\rm spin} = \alpha \sum_{i<j} \vec{s}_i \cdot \vec{s}_j \qquad \alpha = \dfrac{g_s}{\mathcal{V}_{\rm eff}}$

Mean energy per atom: $N\alpha \approx 100 \text{ Hz} = 5\,\text{nK}$

Outline of the talk

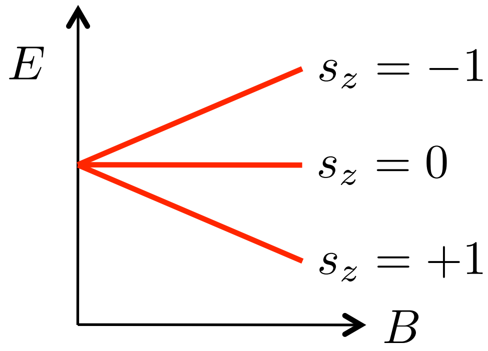
1. Experimental setup and control of magnetisation

2. Phase diagram of a BEC with anti-ferromagnetic interaction (within SMA)

3. The ground state of an anti-ferromagnetic BEC:
A paradigm example of symmetry breaking in Quantum Mechanics

Zeeman effect : linear vs. quadratic contributions

Linear Zeeman effect:

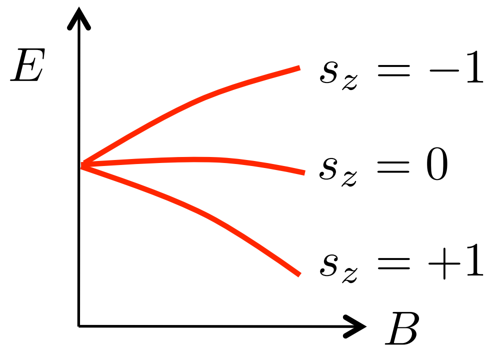


$$V^{(1)}_{\text{Zeeman}} = -\mu B\, S_z = -\mu B (N_{+1} - N_{-1})$$

But S_z is a conserved quantity: $V^{(1)}_{\text{Zeeman}}$ does not contribute to the dynamics of the system

$$|s_z = 0\rangle + |s_z = 0\rangle \quad \longleftrightarrow \quad |s_z = +1\rangle + |s_z = -1\rangle$$

Quadratic Zeeman effect:



$$V^{(2)}_{\text{Zeeman}} = -qB^2 N_0 \ + \ \text{constant}$$

For the F=1 ground state of sodium: $q = 277\ \text{Hz/G}^2$

$q > 0$: favours the accumulation of atoms in $s_z = 0$

⟶ *competition with anti-ferromagnetic interactions*

Ground state of the spin assembly in a magnetic field: a mean-field approach

Minimisation of the energy associated with the Hamiltonian $H = \frac{\alpha}{2} S^2 - \beta N_0$

Trial wave functions: all atoms occupy the same spin state $|\psi\rangle = \begin{pmatrix} \sqrt{n_{+1}}\ \mathrm{e}^{\mathrm{i}\phi_{+1}} \\ \sqrt{n_0}\ \mathrm{e}^{\mathrm{i}\phi_0} \\ \sqrt{n_{-1}}\ \mathrm{e}^{\mathrm{i}\phi_{-1}} \end{pmatrix}$

Second-order phase transition

For $\frac{\beta}{N\alpha} > 1 - \sqrt{1 - M_z^2}$

the three components

$s_z = +1$, $s_z = 0$, $s_z = -1$ co-exist

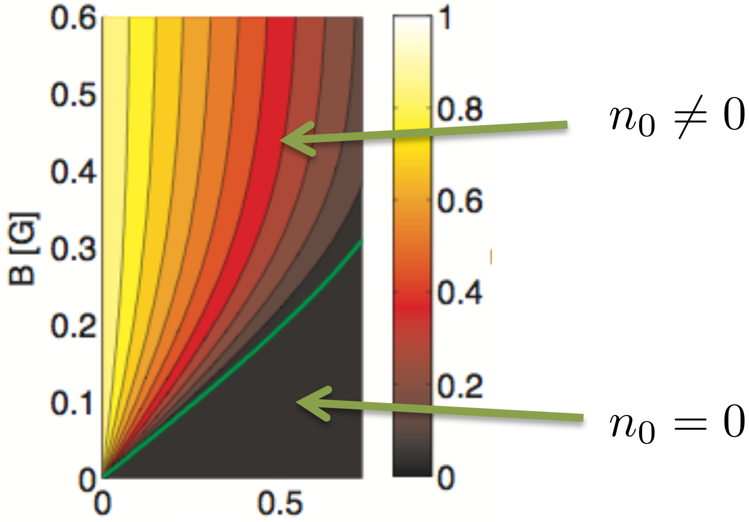
Below this value, only

$s_z = +1$ and $s_z = +1$ are present.

Anti-ferromagnetic interactions dominate

Zhang, Yi, You (2003)

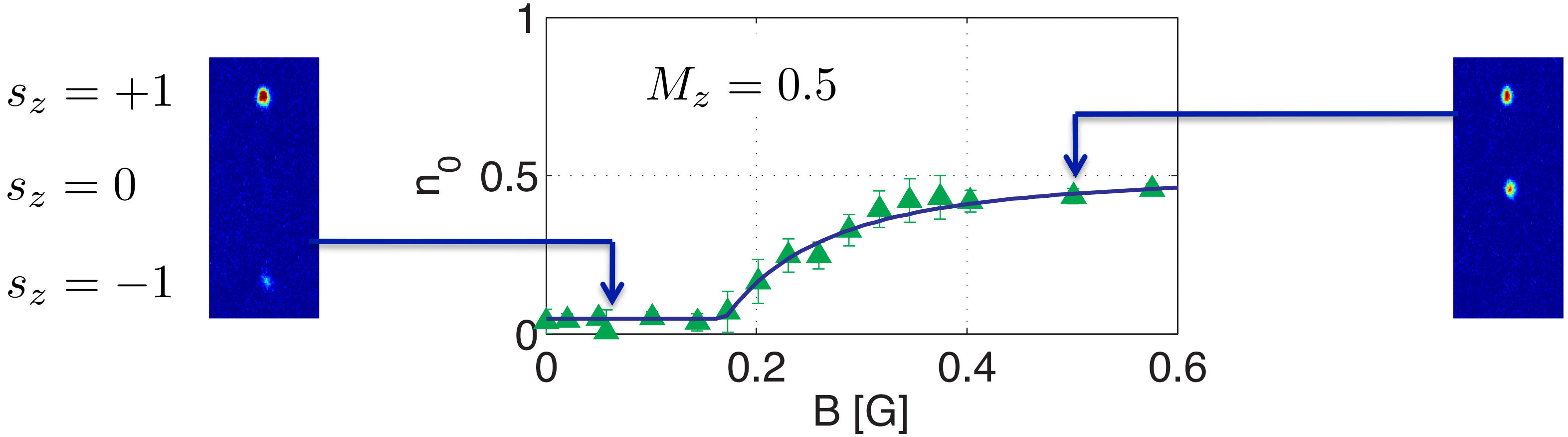
Population n_0 of the s_z=0 state



magnetization M_z

Experimental determination of the phase diagram

Choose a given magnetisation (here M_z=0.5) and measure the fraction n_0 of atoms in s_z=0

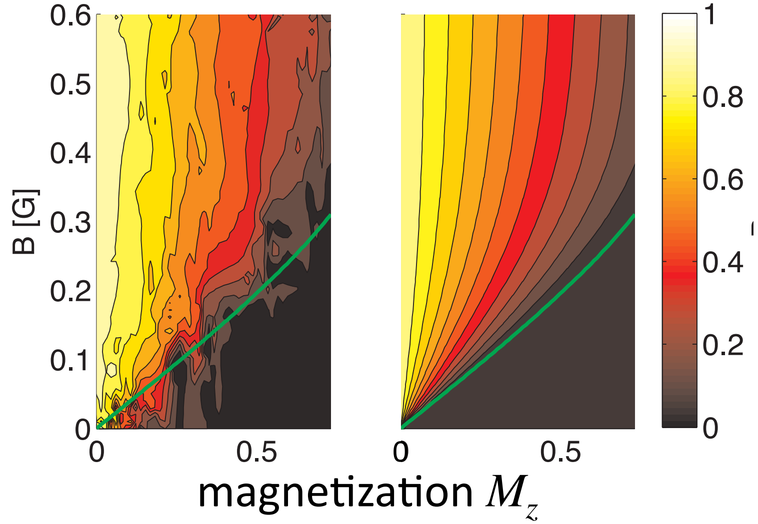


$$|s_z = +1\rangle + |s_z = -1\rangle \quad \longleftrightarrow \quad |s_z = 0\rangle + |s_z = 0\rangle$$

Fraction n_0 for various M_z and magnetic fields

no adjustable parameters

Jacob *et al.*, 2012



Previous measurements:
NIST (2009) for $M_z > 0.5$
Georgia Tech (2011) $\beta > 0$ and < 0, not SMA

Outline of the talk

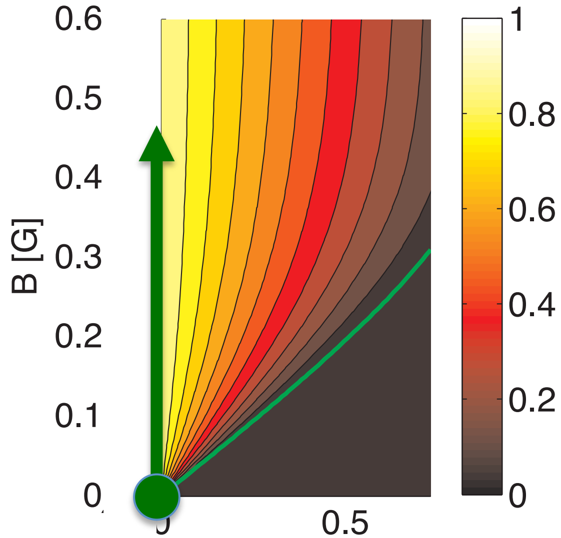
1. Experimental setup and control of magnetisation

2. Phase diagram of a BEC with anti-ferromagnetic interaction (within SMA)

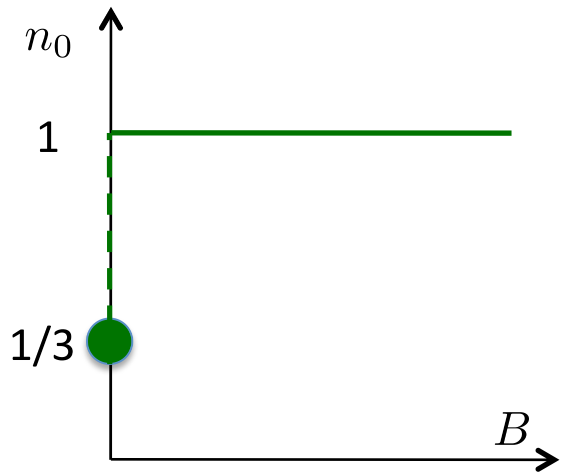
3. The ground state of an anti-ferromagnetic BEC:
   A paradigm example of symmetry breaking in Quantum Mechanics

The remarkable case of zero magnetisation

Population n_0 of the s_z=0 state



magnetization M_z

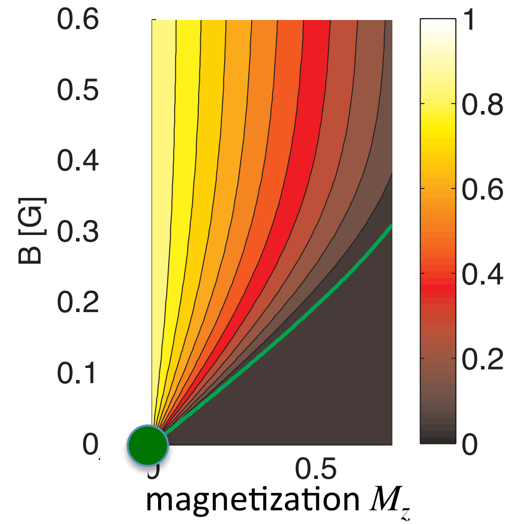


➡ Description of the ground state of the system in the isotropic case $M_z = 0, \; B = 0$

➡ Transition from $n_0 = 1/3$ to $n_0 = 1$ when B increases

The ground state in zero field and zero magnetisation

$$H = \frac{\alpha}{2}\vec{S}^2 + \text{constant}$$



- Exact result (N even): singlet state $|S=0\rangle$

  Unique, breaks no symmetry

- Best approximation (up to $1/N$) within the mean-field ansatz:

$$\rho_{\text{mean field}} = \frac{1}{4\pi}\int |N : s_{\vec{u}} = 0\rangle\langle N : s_{\vec{u}} = 0|\ d^2u$$

Statistical mixture of states where all atoms have a zero spin component (polar state) along an arbitrary direction $\vec{u}$

Each state of the mixture breaks the rotational symmetry

Both approaches lead to $N_0 = \frac{N}{3}$ and $\Delta N_0 = \frac{2}{3\sqrt{5}}N \approx 0.30\ N$

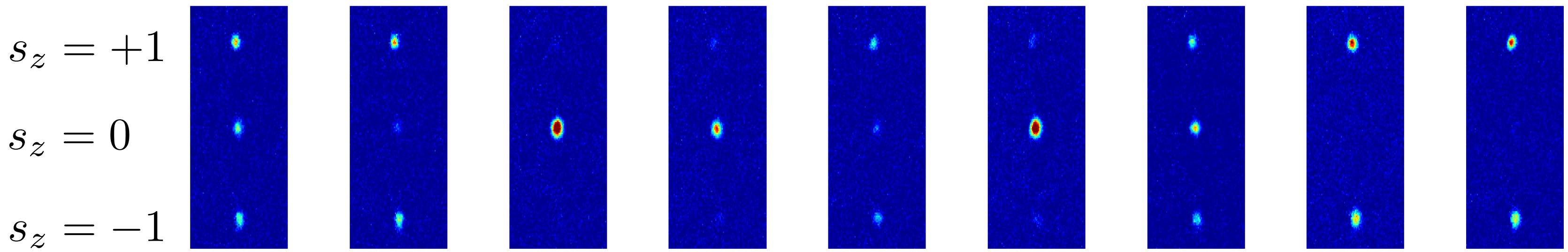
Large (super-Poissonian) fluctuations of N_0 in a Stern & Gerlach measurement along z

These results for N_0 and ΔN_0 still hold at non-zero temperature, provided $k_B T \ll N^2\alpha$

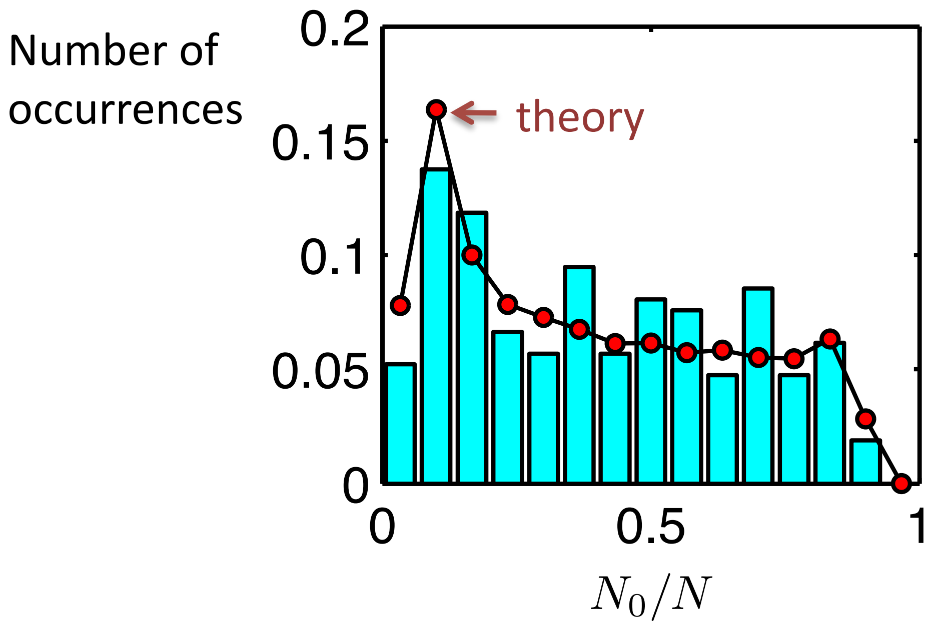
De Sarlo *et al.*, 2013

Experimental investigation of the zero magnetic field case

Series of shots prepared in the same experimental conditions $M_z = 0,\ B = 0$



We observe indeed large (super-Poissonian) fluctuations of N_0 !



Measured standard deviation:

$$\Delta N_0 \approx 0.27N$$

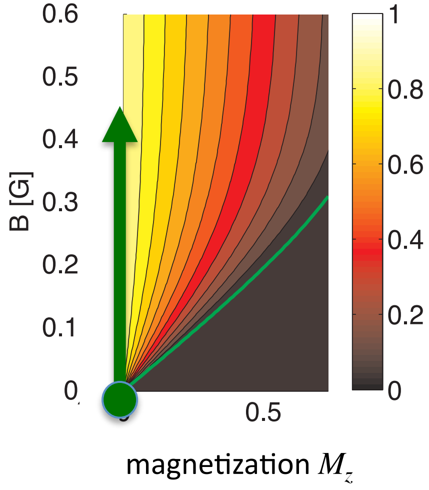
to be compared with the prediction $\Delta N_0 \approx 0.30N$

Fragmented BEC: three macroscopically populated single-particle states

Mueller, Ueda, Baym, Ho (2006)

In the presence of quadratic Zeeman effect: spin thermometry

Population n_0 of the s_z=0 state



Transition from the non polarized state

$$N_0 = \frac{N}{3} \qquad \Delta N_0 \approx 0.30N$$

to the polarized state

$$N_0 = N \qquad \Delta N_0 = 0$$

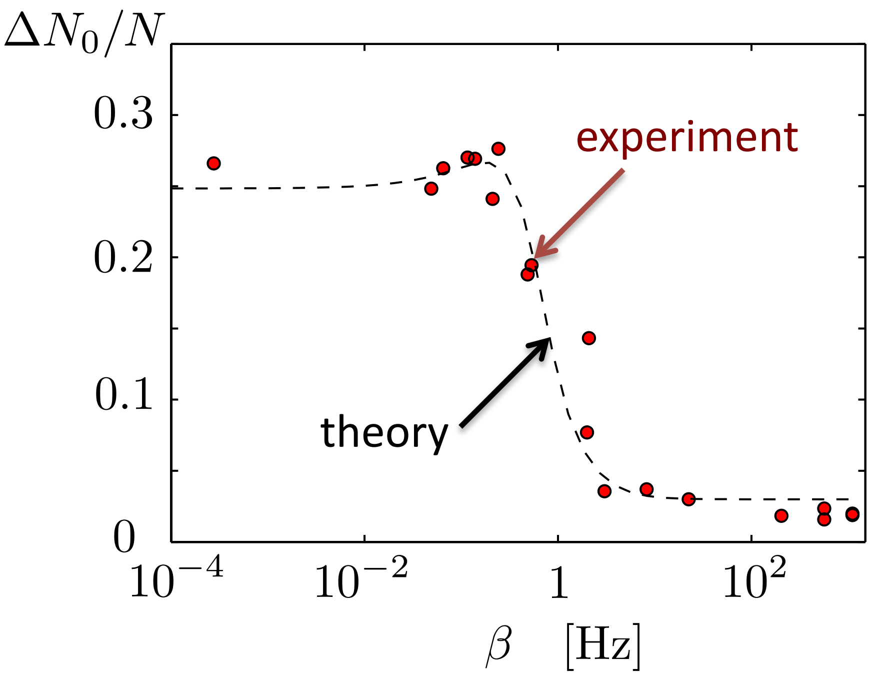
$$H = \frac{\alpha}{2} S^2 - \beta N_0$$

Dotted line: “thermal” mean-field theory
+ non-zero uncondensed fraction

Expected width at Half Maximum

$$\beta \approx 7\ k_B T_{\rm spin}/N \quad \Rightarrow \quad T_{\rm spin} \approx 40\ \rm nK$$

β goes to zero at the thermodynamic limit

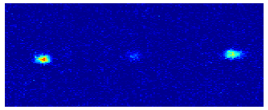


Outlook: symmetry breaking and the measurement problem in QM

Consider again the zero-field & zero-magnetisation case. Two possible descriptions:

$$|S=0\rangle \qquad\qquad \rho_{\text{mean field}} = \frac{1}{4\pi}\int |N:s_{\vec{u}}=0\rangle\langle N:s_{\vec{u}}=0|\; d^2u$$

A given shot of the Stern & Gerlach experiment provides $N_{+1},\ N_0,\ N_{-1}$:



- In the $|S=0\rangle$ point of view, the measurement "builds up" the triplet $\{N_{+1},\ N_0,\ N_{-1}\}$ through a proper use of the usual QM postulates

- In the mean-field (broken symmetry) approach, the system is before any measurement in one randomly chosen $|N:s_{\vec{u}}=0\rangle$ and the measurement "reveals" the value of $\vec{u}$ through $u_z^2 = N_0/N$

N.B.: Both points of view are equivalent (up to 1/N corrections)!

Very similar to the problem of the relative phase of two independent condensates

Ashhab & Leggett

Connection with other symmetry breaking mechanisms

Physical system	Order parameter	Symmetry	Breaking field
Crystal	Position	Translation	Pinning potential
BEC or laser	Macroscopic wave function or electromagnetic field	Phase $\mathrm{U}(1)$	Particle or photon seed
Spin 1 BEC antiferromagnet	Population N_0 of $\lvert s_z = 0\rangle$	Rotation (spin space)	Zeeman effect (quadratic)